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Title

# THE STEADY-STATE SOLUTION OF MULTIPLE PARALLEL CHANNELS IN SERIES AND NON-SERIAL SERVERS WITH BALKING & RENEGING DUE TO LONG QUEUE AND SOME URGENT MESSAGE

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## Abstract:

This paper considers the most appropriate & more general queuing model in respect of customers which are allowed to leave the system at any stage with or without getting service. The paper considers the steady-state behavior of the queuing processes when M service channels in series, are linked with N non-serial channels having balking & reneging phenomenon, wherein:

- Each of M service channels has identical multiple parallel channels.
- Poisson arrivals & exponential service times are followed.
- The service discipline follows SIRO rule (service in random order) instead of FIFO rule (first in first out).
- The customer becomes impatient in the queue after sometime and may leave the system without getting service and can renege due to urgent message.
- Waiting space is infinite.


**Keywords:** Poisson stream, Reneging, Balking, Traffic intensity, Steady-state, Parallel channels, Urgent message.

## Introduction:

The problem of serial queues was studied by O' Brien (1954), Jackson (1954), Barrer (1955), Hunt(1955) and Maggu (1970) in steady-state with Poisson assumptions with the restriction that the customer must go through each service channel before leaving the system. Singh (1984) studied the problem of serial queues introducing the concept of reneging. The steady-state solutions of multiple parallel channels in series with impatient customers are obtained by Singh & Ahuja (1995). The solutions of serial and non-serial queuing processes with reneging and balking phenomenon have been studied by Vikram& Singh (1998). The steady-state solution of serial and non-serial queuing processes with reneging and balking due to long queue and some urgent message and feedback phenomenon is obtained by Singh, Punam & Ashok(2009). In our present society, the impatient customers generate the most appropriate and modern models in the queuing theory. Incorporating this concept, we study the steady-state analysis of general queuing system in the sense that:

- M service channels in series are linked with N non serial channels having reneging and balking phenomenon where each of M service channels has identical multiple parallel channels.
- The input process is Poisson and the service time distribution is exponential.
- The service discipline follows SIRO-rule(service in random order) instead of FIFO- rule (first in first out)
- The customer becomes impatient in queue after sometime and may leave the system without getting service and can renege due to urgent message also.
- The input process depends upon the queue size in non-serial channels.
- Waiting space is infinite.

## Formulation of model:

The system consists of $Q_i$ ( i =1,2,….M) service phases where each service phase $Q_i$ has $c_i$ ( i=1,2,….M) identical parallel service facilities and $Q_{1j}$ channels (j=1,2,….N) with respective servers $S_i$ (i=1,2,……M) and $S_{1j}$ (j=1,2,……N).Customers demanding different types of service arrive from outside the system in Poisson distribution with parameters $\lambda_i$( i =1,2,….M) at $Q_i$ service phase and $\lambda_{1j}$(j=1,2,…..N) at $Q_{1j}$ service phase respectively . But the sight of long queue at $Q_{1j}$ , may discourage the fresh customers from joining it and may decide not to enter the service channel $Q_{1j}$ (j=1,2,….N) then the Poisson input rate $\lambda_{1j}$ would be $\frac{\lambda_{1j}}{m_j+1}$ where $m_j$ is the queue size of $Q_{1j}$ . Further, the impatient customers joining any serial or non-serial service channel $Q_{1j}$ may leave the queue without getting service after a wait of certain time. The service time distribution for the server $S_i$ (i=1,2……,M) and $S_{1j}$ (j=1,2…..,N) are mutually independent negative exponential distribution with $\mu_i (i = 1, 2, ........., M)$ and $\mu_{1j} (j = 1, 2, ...., N)$ respectively. After the completion of service at $Q_i$ (i=1,2,……..M), the customers either leave the system with probability $p_i$ or join the next phase with probability $q_i$ such that $p_i + q_i = 1$ (i= 1,2…..M-1). After completion of service at $Q_M$ , the customers either leave the system with probability $p_M$ or join any of the $Q_{1j}$ (j=1,2,…….,N) with probability $\frac{q_{Mj}}{m_j+1}$ (j=1,2,….,N) such that $p_M + \sum_{j=1}^{N} \frac{q_{Mj}}{m_j+1}$ =1.

If the customers are more than $c_i$ in the $Q_i$ service phase ,all the $c_i$ servers will remain busy and each is putting out the service at mean rate $\mu_i$ and thus the mean service rate at $Q_i$ is $c_i \mu_i$ ,on the other hand if the number of customers is less than $c_i$ in the $Q_i$ service phase ,only $n_i$ out of the $c_i$ servers will be busy and thus the mean service rate at $Q_i$ is $n_i \mu_i$ (i=1,2,…….M).It is assumed that the service commences instantaneously when the customer arrives at an empty service channel.

## Formulation of equations:

Define $P(n_1, n_2, \ldots\ldots n_M ; m_1, m_2, m_3, \ldots\ldots m_N ;t)$ as the probability that at time 't' , there are $n_i$ customers ( which may renege or after being serviced by the $Q_i$ phase either leave the system or join the next service phase ) waiting in the $Q_i$ service phase (i=1,2,……..,M) , $m_j$ customers ( which may renege or after being serviced leave the system ) waiting before the servers $S_{1j}$ (j=1,2,…….N) .

We define the operators $T_{i.}$ and $T_{.i}$ to act upon the vector $\tilde{n} = (n_1, n_2, \ldots\ldots, n_M)$ and $T_{j.}$ and $T_{.j}$ and $T_{.j,\,j+1.}$ to act upon the vector $\tilde{m} = (m_1, m_2, \ldots\ldots, m_N)$ as follows:

$T_{i.}(\tilde{n}) = (n_1, n_2, \ldots\ldots n_i - 1 \ldots n_M)$

$T_{.i}(\tilde{n}) = (n_1, n_2, \ldots\ldots n_i + 1 \ldots n_M)$

$T_{j.}(\tilde{m}) = (m_1, m_2, \ldots\ldots m_j - 1 \ldots m_N)$

$T_{.j}(\tilde{m}) = (m_1, m_2, \ldots\ldots m_j + 1 \ldots m_N)$

$T_{.j,\,j+1.}(\tilde{m}) = (m_1, m_2, \ldots\ldots, m_j + 1, m_{j+1.} - 1 \ldots m_N)$

The customers may leave any service channel without getting service if they receive urgent call while waiting.

$$\frac{dP(\tilde{n},\tilde{m};t)}{dt} = -\left[\begin{array}{l} \sum_{i=1}^{M}\lambda_i + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j+1} + \sum_{i=1}^{M}\delta(n_i)\{\mu_{in_i} + \delta_{n_i-c_i}(\alpha_i + r_{in_i})\} \\ + \sum_{j=1}^{N}\delta(m_j)\{(\mu_{1j}) + \beta_j + R_{jm_j}\} \end{array}\right] P(\tilde{n},\tilde{m};t)$$

$$+ \sum_{i=1}^{M}\lambda_i P(T_{i.}(\tilde{n}),\tilde{m};t)) + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j}P(\tilde{n};T_{j.}(\tilde{m});t)$$

$$+ \sum_{i=1}^{M}\delta_{n_i-c_i}(\alpha_i + r_{in_i})P(T_{.i}(\tilde{n}),\tilde{m};t) + \sum_{i=1}^{M-1}q_i\mu_{in_{i+1}}P(T_{.i\,i+1.}(\tilde{n}),\tilde{m};t) +$$

$$\sum_{i=1}^{M}p_i\mu_{in_{i+1}}P(T_{.i}(\tilde{n}),\tilde{m};t) + \sum_{j=1}^{N}\mu_{Mn_M+1}\frac{q_{Mj}}{m_j}P(n_1,n_2,.....n_M+1,T_{j.}(\tilde{m});t)$$

$$+ \sum_{j=1}^{N}(\mu_{1j} + \beta_j + R_{jm_j+1})P(\tilde{n};T_{.j}(\tilde{m});t) \qquad \text{.................................}(1)$$

Following the procedure given by Kelly(1979), we write difference differential equations :

for $n_i \geq 0$ , $mj \geq 0$ , (i = 1,2,….M ) , ( j= 1,2….N) .

Where

$$\delta(x) = \begin{bmatrix} 1 & when & x \neq 0 \\ 0 & when & x = 0 \end{bmatrix}$$

$$\delta(n_i) = \begin{bmatrix} 0 & when & n_i = 0 \\ 1 & when & n_i \neq 0 \end{bmatrix}$$

$$\delta_{(n-c)} = \begin{bmatrix} 0 & when & n < c \\ 1 & when & n \geq c \end{bmatrix}$$

$$\mu_{in_i} = \begin{bmatrix} n_i\mu_i & when & 1 \leq n_i < c_i \\ c_i\mu_i & when & n_i \geq c_i \end{bmatrix}$$

$$r_{in_i} = \mu_i e^{\frac{-\mu_i T_{0i}}{n_i}} \Big/ \left(1 - e^{\mu_i e^{\frac{-\mu_i T_{0i}}{n_i}}}\right) \qquad ; i = 1, 2, \ldots\ldots M$$

$$R_{jm_j} = \mu_{1j} e^{\frac{-\mu_{1j} T_{0j}}{m_j}} \Big/ 1 - e^{\frac{\mu_{1j} T_{0j}}{m_j}}$$

Where $r_{in_i}$ *and* $R_{jm_j}$ are the average rates at which the customers renege after a wait of certain time $T_{0i}$ *and* $T_{0j}$ whenever there are $n_i$ and $m_j$ customers in the $Q_i$ and $Q_{1j}$ service phases respectively . $\alpha_i$ and $\beta_j$ are the mean reneging rates at serial and non-serial service phases due to some urgent message and $P(\tilde{m},\tilde{n};t) = 0$ if any of the arguments is negative

## Steady-State Equations:

We write the steady-state equations of the queuing model by equating the time derivative to zero in the equation (1)

$$\left[\sum_{i=1}^{M}\lambda_i + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j+1} + \sum_{i=1}^{M}\delta(n_i)\{\mu_{in_i} + \delta_{n_i-c_i}(\alpha_i + r_{in_i})\} + \sum_{j=1}^{N}\delta(m_j)\{(\mu_{1j}) + \beta_j + R_{jm_j}\}\right] P(\tilde{n},\tilde{m})$$

$$= \sum_{i=1}^{M}\lambda_i P(T_{i.}(\tilde{n}),\tilde{m})) + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j} P(\tilde{n};T_{j.}(\tilde{m}))$$

$$+ \sum_{i=1}^{M}\delta_{n_i-c_i}(\alpha_i + r_{in_i}) P(T_{.i}(\tilde{n}),\tilde{m}) + \sum_{i=1}^{M-1} q_i \mu_{in_{i+1}} P(T_{.i\,i+1.}(\tilde{n}),\tilde{m})$$

$$+ \sum_{i=1}^{M} p_i \mu_{in_{i+1}} P(T_{.i}(\tilde{n}),\tilde{m}) + \sum_{j=1}^{N}\mu_{Mn_M+1}\frac{q_{Mj}}{m_j} P(n_1, n_2, \ldots..n_M + 1, T_{j.}(\tilde{m}))$$

$$+ \sum_{j=1}^{N}(\mu_{1j} + \beta_j + R_{jm_j+1}) P(\tilde{n};T_{.j}(\tilde{m})) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

for $n_i \geq 0$ , $mj \geq 0$ ; (i = 1,2,….,M ) ; ( j= 1,2,….,N) .

**Case I:– When $n_i < c_i$**

For $n_i < c_i$ , the resulting equations (2) reduce to as under:

$$\left[\sum_{i=1}^{M}\lambda_i+\sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j+1}+\sum_{i=1}^{M}\delta(n_i)(n_i\mu_i)+\sum_{j=1}^{N}\delta(m_j)\{(\mu_{1j})+\beta_j+R_{jm_j}\}\right]P(\tilde{n},\tilde{m})$$

$$=\sum_{i=1}^{M}\lambda_i P(T_{i.}(\tilde{n}),\tilde{m}))+\sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j}P(\tilde{n};T_{j.}(\tilde{m}))$$

$$+\sum_{i=1}^{M-1}q_i\mu_i(n_i+1)P(T_{.i\,i+1.}(\tilde{n}),\tilde{m})$$

$$+\sum_{i=1}^{M}p_i\mu_i(n_i+1)P(T_{.i}(\tilde{n}),\tilde{m})+\sum_{j=1}^{N}\mu_M(n_M+1)\frac{q_{Mj}}{m_j}P(n_1,n_2,.....n_M+1,T_{j.}(\tilde{m}))$$

$$+\sum_{j=1}^{N}(\mu_{1j}+\beta_j+R_{jm_j+1})P(\tilde{n};T_{.j}(\tilde{m})) \qquad ..............................................(3)$$

The solutions of the steady state equations (3) can be verified to be:

$$P(\tilde{n},\tilde{m})=P(\tilde{0},\tilde{0})\left[\left(\frac{1}{\lfloor n_1}\right)\left(\frac{\lambda_1}{\mu_1}\right)^{n_1}\right]\left[\left(\frac{1}{\lfloor n_2}\right)\left(\frac{\lambda_2+q_1\alpha_1'}{\mu_2}\right)^{n_2}\right]\left[\left(\frac{1}{\lfloor n_3}\right)\left(\frac{\lambda_3+q_2\alpha_2'}{\mu_3}\right)^{n_3}\right]$$

$$.............\left[\left(\frac{1}{\lfloor n_M}\right)\left(\frac{\lambda_M+q_{M-1}\alpha_{M-1}'}{\mu_M}\right)^{n_M}\right]\left[\left(\frac{1}{\lfloor m_1}\right)\left(\frac{(\lambda_{11}+\mu_M q_{M1}\rho_M)^{m_1}}{\prod_{j=1}^{m_1}(\mu_{11}+\beta_1+R_{1j})}\right)\right]$$

$$\left[\left(\frac{1}{\lfloor m_2}\right)\left(\frac{(\lambda_{12}+\mu_M q_{M2}\rho_M)^{m_2}}{\prod_{j=1}^{m_2}(\mu_{12}+\beta_2+R_{2j})}\right)\right]................\left[\left(\frac{1}{\lfloor m_N}\right)\left(\frac{(\lambda_{1N}+\mu_M q_{MN}\rho_M)^{m_N}}{\prod_{j=1}^{m_N}(\mu_{1M}+\beta_M+R_{Mj})}\right)\right] \qquad .......(4)$$

for

$n_i \geq 0$ , $mj \geq 0$ ; ($i$ = 1,2,….,M ) ; ( $j$= 1,2,….,N) .

where

$$\rho_M = \frac{\lambda_M + q_{M-1}\alpha'_{M-1}}{\mu_M}$$

$$\alpha'_1 = \lambda_1$$

$$\alpha'_k = \lambda_k + q_{k-1}\alpha'_{k-1} \qquad k = 2,3,........,M-1$$

**Case II:– When $n_i \geq c_i$**

For $n_i \geq c_i$ , the resulting equations (2) will reduce to as under:

$$\left[\sum_{i=1}^{M}\lambda_i + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j+1} + \sum_{i=1}^{M}\delta(n_i)(c_i\mu_i + \alpha_i + r_{in_i}) + \sum_{j=1}^{N}\delta(m_j)\{(\mu_{1j}) + \beta_j + R_{jm_j}\}\right]P(\tilde{n},\tilde{m})$$
$$= \sum_{i=1}^{M}\lambda_i P(T_{i.}(\tilde{n}),\tilde{m})) + \sum_{j=1}^{N}\frac{\lambda_{1j}}{m_j}P(\tilde{n};T_{j.}(\tilde{m})) + \sum_{i=1}^{M}(p_i c_i \mu_i + \alpha_i + r_{in_{i+1}})P(T_{.i}(\tilde{n}),\tilde{m})$$
$$+ \sum_{i=1}^{M-1} q_i c_i \mu_i P(T_{.i\,i+1.}(\tilde{n}),\tilde{m}) + \sum_{j=1}^{N}\mu_M c_M \frac{q_{Mj}}{m_j}P(n_1,n_2,.....n_M+1,T_{j.}(\tilde{m}))$$
$$+ \sum_{j=1}^{N}(\mu_{1j} + \beta_j + R_{jm_j+1})P(\tilde{n};T_{.j}(\tilde{m})) \qquad ........................................(5)$$

The solutions of the steady-state equations can be verified to be:

$$P(\tilde{n},\tilde{m}) = P(\tilde{0},\tilde{0}) \left[\frac{(\lambda_1)^{n_1}}{\prod_{i=1}^{n_1}(c_1\mu_1 + \alpha_1 + r_{1i})}\right]\left[\frac{\{\lambda_2(c_1\mu_1 + \alpha_1 + r_{1n_1+1}) + c_1 q_1 \mu_1 \alpha_1\}^{n_2}}{\prod_{i=1}^{n_2}(c_2\mu_2 + \alpha_2 + r_{2i})(c_1\mu_1 + \alpha_1 + r_{1n_1+1})^{n_2}}\right]$$
$$\left[\frac{\{\lambda_3\prod_{i=1}^{2}(c_i\mu_i + \alpha_i + r_{in_1+1}) + c_2 q_2 \mu_2 \alpha_2\}^{n_3}}{\prod_{i=1}^{n_3}(c_3\mu_3 + \alpha_3 + r_{3i})\left[\prod_{i=1}^{2}(c_i\mu_i + \alpha_i + r_{in_1+1})\right]^{n_3}}\right] ..........................................$$
$$\left[\frac{\{\lambda_M\prod_{i=1}^{M-1}(c_i\mu_i + \alpha_i + r_{in_i+1}) + c_{M-1}q_{M-1}\mu_{M-1}\alpha_{M-1}\}^{n_M}}{\prod_{i=1}^{n_M}(c_M\mu_M + \alpha_M + r_{Mi})\left[\prod_{i=1}^{M-1}(c_i\mu_i + \alpha_i + r_{in_i+1})\right]^{n_M}}\right]\left[\frac{(\lambda_{11} + \mu_M c_M \rho'_M q_{M1})^{m_1}}{\lfloor m_1 \prod_{j=1}^{m_1}(\mu_{11} + \beta_1 + R_{1j})}\right]$$
$$\left[\frac{(\lambda_{12} + \mu_M c_M \rho'_M q_{M2})^{m_2}}{\lfloor m_2 \prod_{j=1}^{m_2}(\mu_{12} + \beta_2 + R_{2j})}\right] .............................. \left[\frac{(\lambda_{1N} + \mu_M c_M \rho'_M q_{MN})^{m_N}}{\lfloor m_N \prod_{j=1}^{m_N}(\mu_{1M} + \beta_M + R_{Mj})}\right] \qquad (6)$$

$$\text{Where} \quad \rho_m^{'} = \frac{\lambda_M \prod_{i=1}^{M-1}(c_i\mu_i + \alpha_i + r_{in_i+1}) + c_{M-1}\mu_{M-1}q_{M-1}\alpha_{M-1}}{(c_M\mu_M + \alpha_M + r_{Mn_M+1})\prod_{i=1}^{M-1}(c_i\mu_i + \alpha_i + r_{in_i+1})} \quad i$$

$$\alpha_1 = \lambda_1$$

$$\alpha_k = \lambda_k \prod_{i=1}^{k-1}(c_i u_i + \alpha_i + r_{in_i+1}) + q_{k-1}\alpha_{k-1}u_{k-1}c_{k-1}; \quad k = 2,3,......., M-1$$

We obtain $P(\tilde{0},\tilde{0})$ from (4) and (6) by the normalizing condition $\sum_{\tilde{m}=\tilde{0}}^{\infty}\sum_{\tilde{n}=\tilde{0}}^{\infty} P(\tilde{n},\tilde{m}) = 1$ and with the restrictions that traffic intensity of each service channel of the system is less than unity Thus $P(\tilde{n},\tilde{m})$ is completely determined.

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